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RATE OF GROWTH OF AIDS EPIDEMIC IN EUROPE: A COMPARATIVE ANALYSIS

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Estimates of the rate of increase of the AIDS epidemic for each of 18 European countries are obtained by fitting a Poisson process with exponential rate of growth to data. A linear regression model of these estimates on the proportion of cases that are intravenous drug users, homosexuals/bisexuals and heterosexuals, was estimated and suggested that the rates of growth of the epidemics amongst these groups are different and in increasing order. Empirical Bayes estimates of the rates are obtained for each country.

1. Introduction

The work in this paper has a two-fold aim. One is to compare the incidence of AIDS in the different European countries and the other is to explore the use of empirical Bayes methods for improving prediction of the growth of the epidemic, especially for the U.K. The key idea of the latter is that under certain plausible assumptions the best estimate of a parameter describing the U.K. experience, for example, is a weighted combination of the estimate from the U.K. data, and of that from other countries where the situation is similar. The gain from using the empirical Bayes approach is small so the main emphasis of the paper is on the former aim.

The data are summarized in table 1, showing the numbers of AIDS cases reported quarterly in 1986 and 1987, and total numbers reported before 1986. They are taken from a report of the World Health Organization (WHO) collaborating centre on AIDS. Countries reporting very few cases have been omitted. These include Bulgaria, East Germany, Hungary, Iceland, Luxemburg, Malta, Poland, Romania and the U.S.S.R.

One possibility is to analyse the total numbers of cases per million of population. This shows enormous variation between countries. It is likely, however, that the growth rate of the epidemic, as measured by an effective doubling time, is much more stable; therefore, we have concentrated on the analysis of the growth rate.

2. Initial analysis

Whereas there is strong evidence, especially for the U.K., of a slowing down from exponential growth for the spread of AIDS, it is reasonable to postulate a Poisson process of exponentially increasing rate in order to model the results presented in table 1; i.e. to suppose for each country that the rate of occurrence of cases at time t is

$$\alpha e^{\beta t},$$

where α and β are different for each country. In view of the remarks above we concentrate on the study of β. The emphasis is not on extrapolation but rather on summarizing the data.

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Table 1. Total number of AIDS cases reported in 18 European countries

country	Mar. 1986	Jun. 1986	Sep. 1986	Dec. 1986	Mar. 1987	Jun. 1987	Sep. 1987	Dec. 1987
Austria	34	36	44	54	72	93	120	139
Belgium	160	171	180	207	230	255	277	277
Denmark	80	93	107	131	150	176	202	228
Finland	11	11	14	14	19	19	22	24
France	707	859	1050	1221	1632	1980	2532	3073
F.R.G.	459	538	675	826	999	1133	1400	1669
Greece	14	22	25	35	42	49	78	88
Ireland	8	9	10	12	14	19	19	25
Israel	23	24	31	34	38	39	43	47
Italy	219	300	367	523	664	870	1104	1411
Netherlands	120	146	180	218	260	308	370	420
Norway	21	24	26	35	45	49	64	70
Portugal	24	28	40	46	54	67	81	90
Spain	145	177	201	264	357	508	624	789
Sweden	50	57	76	90	105	129	143	163
Switzerland	113	138	170	192	227	266	299	355
U.K.	287	340	512	610	729	870	1067	1227
Yugoslavia	3	3	3	8	10	11	21	26

The model specifies that for a particular country the numbers of AIDS cases, N_j, in (t_{j-1}, t_j) $(j = 0, \ldots, s)$, with $t_{-1} = -\infty$, follow Poisson distributions with means

$$\int_{-\infty}^{t_0} \alpha \exp(\beta t)\, \mathrm{d}t = \alpha \exp(\beta t_0)/\beta = \alpha\omega_0(\beta),$$

$$\int_{t_{j-1}}^{t_j} \alpha \exp(\beta t)\, \mathrm{d}t = \alpha \exp(\beta t_j)\,(1-\mathrm{e}^{-\beta d_j})/\beta = \alpha\omega_j(\beta),$$

for $j = 1, \ldots, s$, where $d_j = t_j - t_{j-1}$.

Table 2. Estimated exponential rates of increase for 18 European countries

(Based on report of the WHO Collaborating Centre on AIDS. Doubling time is $0.693/\beta$ yr. est., estimated; emp., empirical; IDU., intravenous drug users; HE., heterosexuals; HOM., homosexuals/bisexuals; s.e., standard error.)

	est. $\hat{\beta}$ (per year)		fitted $\hat{\beta}$* from (3) (per year)		emp. Bayes (per year)				
	(*a*) est.	(*b*) s.e.	(*c*) est.	(*d*) s.e.	(*e*) est.	(*f*) s.e.	% IDU (*g*)	% HE. (*h*)	% HOM. (*i*)
Austria	0.834	0.081	0.807	0.041	0.823	0.062	23	2	5
Belgium	0.304	0.028	0.416	0.111	0.313	0.027	2	58	25
Denmark	0.585	0.048	0.661	0.046	0.601	0.043	2	5	84
Finland	0.447	0.124	0.618	0.041	0.553	0.076	4	17	71
France	0.851	0.017	0.724	0.038	0.847	0.017	12	5	62
F.R.G.	0.731	0.021	0.714	0.042	0.731	0.021	9	3	75
Greece	0.985	0.116	0.571	0.848	0.741	0.074	1	23	47
Ireland	0.803	0.161	0.846	0.045	0.834	0.083	30	3	27
Israel	0.394	0.081	0.684	0.052	0.514	0.062	2	0	61
Italy	1.033	0.030	1.054	0.094	1.035	0.029	64	4	21
Netherlands	0.687	0.040	0.687	0.048	0.687	0.037	4	2	87
Norway	0.681	0.098	0.677	0.041	0.679	0.069	6	7	79
Portugal	0.710	0.089	0.547	0.064	0.636	0.066	6	35	51
Spain	0.997	0.039	0.999	0.076	0.997	0.036	53	1	25
Sweden	0.641	0.061	0.639	0.047	0.641	0.052	0	7	81
Switzerland	0.636	0.041	0.744	0.035	0.652	0.038	19	10	63
U.K.	0.717	0.024	0.665	0.047	0.714	0.024	2	4	85
Yugoslavia	1.317	0.273	0.810	0.042	0.867	0.091	28	8	40

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY B BIOLOGICAL SCIENCES

Maximum likelihood estimates of α and β are the solutions of

$$\hat{\alpha} = \sum n_j / \sum \omega_j(\hat{\beta}), \tag{1}$$

$$\sum n_j \omega_j'(\hat{\beta})/\omega_j(\hat{\beta}) = (\sum n_j)\,[\sum \omega_j'(\hat{\beta})]\,[\sum \omega_j(\hat{\beta})]^{-1}. \tag{2}$$

Note that (2) is independent of the overall incidence rate $\hat{\alpha}$. This is related to the fact that, conditional on the total number of cases $\sum N_j$, $(N_0, \ldots, N_s)$ have a multinomial distribution with probabilities $\omega_j(\beta)/\sum \omega_k(\beta)$, not involving α. The maximum likelihood estimate of β from the conditional distribution satisfies (2).

The variances, $\hat{v}_\beta$ of the resulting estimates can be calculated via standard methods.

Table 2 (columns (*a*) and (*b*)) shows the estimates of $\hat{\beta}$ and their standard errors. Note that $\hat{\beta}$ is in yr^{-1} and that the population doubling time is $0.69/\beta$ yr.

3. Further analysis

Most of the estimates in table 2 lie between 0.6 and 1 and although considering the nature of the data, this is not a very wide range, the differences are highly significant statistically and large enough to be of genuine importance. It is therefore necessary to try to explain as much of the variation as possible in systematic terms. The only basis we have for this lies in the proportions of cases of various types, as summarized in the right-hand section of table 2.

Note that:

(i) Belgium has a very low rate of increase in the number of AIDS cases. It also has a high proportion (58%) of heterosexual cases and known special circumstances. This is however some indication that a heterosexual (HE) epidemic would grow more slowly than one driven from the other sources.

(ii) The Mediterranean countries except for Israel have high values of β and also high proportions of intravenous drug users (IDUS).

Maximum likelihood estimates of the fitting of a normal model with mean

$$\beta = \theta_0 + \theta_1(\%\ \text{IDU}) + \theta_2(\%\ \text{HE}), \tag{3}$$

and variance

$$\tau^2 + \hat{v}_\beta.$$

gave

parameter	estimate (per year)	s.e. (per year)
θ_0	67.14	5.47
θ_1	0.62	0.19
θ_2	−0.46	0.23

and estimate of τ as 0.097 (yr^{-1}). The fitted rates of increase for each country and corresponding standard errors are displayed in table 2 (columns (*c*) and (*d*)).

The estimated model suggests, but of course does not prove, that the rates of growth of the epidemics amongst heterosexual, homosexual/bisexual and IDUS are different and in increasing order. A direct check of this hypothesis has not been attempted here.

Clearly, Belgium has a major effect on the inclusion of the proportion of heterosexuals in (3). The fitting of the model with Belgium deleted from the data gives

$$100\hat{\beta}^* = 64.80 + 0.64\ (\%\ \text{IDU}),$$

where the coefficient 0.64 has standard error 0.15.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY B BIOLOGICAL SCIENCES

Note that omission of percentage of heterosexuals (% HE) has had little effect on the coefficient of percentage of intravenous drug users (% IDU) providing some reinforcement of the suggestion that an epidemic among IDUs would have a shorter doubling time than one among homosexuals.

4. Empirical Bayes estimation

To improve the estimate of the rate of increase for a particular country, by combining the information from all the countries, we may calculate the empirical Bayes estimate of β_i and its standard error. This essentially combines the direct estimate for the country with information from the remaining countries. For the latter we use the regression estimate from the previous section. The relative weights to be attached to the two sources of information depend on the relative magnitudes of the variance of the individual estimate and of the scatter about the regression.

In fact, the Empirical Bayes estimate of β_i, $\beta_i^{\mathrm{EB}} = \lambda_i \hat{\beta}_i + (1-\lambda_i)\ \hat{\beta}_i^*$ with standard error $(\lambda_i \hat{v}_{\beta_i})^{\frac{1}{2}}$ where $\lambda_i = \tau^2/(\tau^2 + \hat{v}_{\beta_i})$. This provides a compromise between the individual estimate $\hat{\beta}_i$ and the estimate $\hat{\beta}_i^*$ from the fitting of the 'regression' (3).

Note that the estimate directly from the U.K. is not very different from the regression estimate. Thus even if the real precision of the U.K. estimate is much lower than it purports to be there would be little gain for the U.K. from the use of empirical Bayes ideas. For some other countries, especially those for which the amount of 'local' information is small, the use of empirical Bayes methods does induce an appreciable change in the estimated parameters.

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